55.

$$\frac{dy}{dx} = e^{\frac{x-y}{2}} \qquad -\frac{1}{e^{\frac{x-y}{2}}} = -e^{-\left(\frac{x-y}{2}\right)}$$

$$= -e^{\frac{-x+y}{2}}$$

$$\frac{dy}{dx} = -e^{\frac{y-x}{2}} \longleftarrow$$

if $\ell_1 \perp \ell_2$ then $m_1 = -\frac{1}{m_2}$

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65 $\frac{dy}{dx} = x - \frac{1}{x^2}$ $y(1) = 2$

a) $\frac{dy}{dx} = x - x^{-2}$

$$y = \frac{x^2}{2} - \frac{x^{-1}}{-1} + c$$

$$2 = \frac{1}{2} + 1 + c$$

$$\frac{1}{2} = c$$

$$y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2}$$

b) $y(-1) = 1$

$$1 = \frac{1}{2} - 1 + c$$

$$1\tfrac{1}{2} = c$$

$$y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2}$$

c) $$y = \begin{cases} \frac{x^2}{2} + \frac{1}{x} + C_1 & x<0 \\ \frac{x^2}{2} + \frac{1}{x} + C_2 & x>0 \end{cases}$$

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6.2 antiderivatives by substitution

memorize basic formulas p 332

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \qquad \int x^3\,dx = \frac{x^4}{4} + c$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + c \qquad \int (\sin x)^3 \cos x\,dx$$

$u = f(x)$

let $u = \sin x$ $du = \cos x\,dx$

$$\int u^3\,du = \frac{u^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

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$$\int e^{\tan x} \sec^2 x\,dx$$

$u = \tan x$ $du = \sec^2 x\,dx$

$$\int e^u\,du = e^u + c$$

$$= e^{\tan x} + c$$

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$$\frac{1}{6}\int 6x^2\sqrt{5+2x^3}\,dx$$

let $u = ?\ \ 5+2x^3$

$du = 6x^2 \cdot dx$

solve for dx

$dx = \frac{du}{6x^2}$

$$\int \cancel{x^2}\sqrt{u}\,\frac{du}{6\cancel{x^2}}$$

$$\frac{1}{6}\int \sqrt{u}\,du$$

$$\frac{1}{6}\int \sqrt{5+2x^3}\;6x^2\,dx$$

$$\frac{1}{6}\int \sqrt{u}\,du = \frac{1}{6}\frac{u^{3/2}}{3/2} = \frac{1}{6}\frac{2}{3}(5+2x^3)^{3/2}+c$$

$\sqrt{u} = u^{1/2}$

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$$\int \cot(7x)\,dx$$

$$\frac{1}{7}\int 7\frac{\cos(7x)}{\sin(7x)}\,dx = \int \frac{\cancel{\cos(7x)}}{u}\,\frac{du}{7\cancel{\cos(7x)}}$$

let $u = \sin(7x)$

$du = 7\cos(7x)\,dx$

solve for dx: $dx = \frac{du}{7\cos(7x)}$

$$\frac{1}{7}\int \frac{1}{u}\,du$$

$$\frac{1}{7}\ln|u| + c$$

$$\frac{1}{7}\ln|\sin(7x)| + c$$

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$$\frac{1}{2}\int_0^1 \frac{2x}{x^2-4}\,dx = \frac{1}{2}\int_{-4}^{-3}\frac{du}{u} = \frac{1}{2}\ln|u|\Big|_{-4}^{-3}$$

$u = x^2-4$

$du = 2x\,dx$

if $x=0$ $u=-4$

$x=1$ $u=-3$

$$= \frac{1}{2}\ln|-3| - \frac{1}{2}\ln|-4|$$

$$= \frac{1}{2}(\ln 3 - \ln 4)$$

$$= \frac{1}{2}\ln\frac{3}{4}$$

$$= \ln\left(\frac{3}{4}\right)^{\frac{1}{2}} = \ln\sqrt{\frac{3}{4}}$$

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